LITERATURE CITED

1. M. V. Vedernikov, V. G. Dvunitkin, and A. Zhumagulov, Fiz. Tverd. Tela, 20, No. 11, 3302-3305 (1978).
2. I. I. Novikov, Surveys on the Thermophysical Properties of Substances [in Russian], No. 3, Moscow (1977), pp. 3-42.
3. I. I. Novikov, Surveys on the Thermophysical Properties of Substances [in Russian], No. 1, Moscow (1980), pp. 9-36.
4. G. N. Dul'nev and V. V. Novikov, Inzh.-Fiz. Zh., 33, No. 3, 439-443 (1977).
5. G. N. Dul'nev and Yu. P. Zarichnyak, Forecasting the Thermophysical, Electrophysical, and Physicomechanical Properties of Mixtures and Composites: Preprint of a Paper at the Fifth European Conference on the Thermophysical Properties of Solids [in Russian], Moscow (1976).
6. Yu. P. Zarichnyak and T. A. Lisnenko, Inzh.-Fiz. Zh., 33, No. 4, 642-647 (1977).
7. E. M. Savitskii, V. F. Terekhova, Yu. P. Zarichnyak, et al., Proceedings of the Third All-Union Conference on Alloys of Rare and Refractory Metals Having Special Physical Properties [in Russian], Moscow (1979), pp. 134-136.
8. V. F. Glushkova, Yu. P. Zarichnyak, T. A. Lisnenko, and P. A. Tikhonov, A Study of the Properties of Solid Solutions of Rare-Earth Oxides [in Russian], Moscow (1977), Dep. VINITI, No. 2963-77.
9. A. A. Basov, Yu. P. Zarichnyak, and T. A. Lisnenko, Teplofiz. Vys. Temp., 15, No. 4, 918-926 (1977).
10. G. N. Dul'nev, Inzh.-Fiz. Zh., 9, No. 3, 399-404 (1965).
11. D. P. Volkov, G. N. Dul'nev, Yu. P. Zarichnyak, and B. L. Muratova, Inzh.-Fiz. Zh., 46, No. 2, 247-252 (1984).
12. G. Schulze, Metal Physics [Russian translation], Moscow (1971).
13. A. Zhumagulov and M. V. Vedernikov, Fiz. Tverd. Tela, 22, 892-894 (1980).

PARAMETERS OF A COUNTERCURRENT THERMAL-DIFFUSION APPARATUS WITH FLOW CLOSURE AND SAMPLING

G. D. Rabinovich and A. V. Suvorov

UDC 621.039.341.6

Formulas have been derived for the optimum apparatus parameters.

An apparatus has been described [1] based on a planar thermal-diffusion column, with the mixture pumped in opposite directions at the ends. A closed loop can be used (Fig. 1b) or partial or complete product tapoff (Fig. 1a and c). The apparatus and the calculation method [2] were devised for separating petroleum oil fractions. One can evaluate the performance in separating other liquid or gas mixtures under laboratory or plant conditions by means of a more general method, which has been used in considering apparatus with complete product removal (Fig. 1c) [3, 4] or with complete product return (Fig. 1b) [5]. Our purpose is a theoretical analysis of the case where part of the material is returned (Fig. 1a).

We consider the stationary separation of a binary liquid mixture, where the concentration changes occur in the range allowing of the approximation $c(1-c) \approx a + bc$, which includes important cases such as removing a minor impurity when the content of the main component is high ($c(1-c) \approx 1-c$) and enrichment when there is only a low content of the main component ($c(1-c) \approx c$). The model [3-5] shows that the target component passes through the separating region:

$$\tau = H\left[a + bc - \frac{dc}{dy}\right], \quad (1)$$

Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 53, No. 2, pp. 280-284, August, 1987. Original article submitted May 10, 1986.

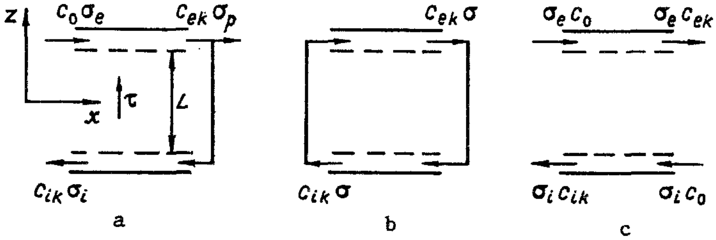

Fig. 1. Scheme for supplying and removing mixture: a) tapoff with flow closure; b) closed system without tapoff; c) countercurrent without closure.

with the transport in the stationary state constant and the same for any horizontal section, so (1) can be differentiated with respect to y to get an equation for the concentration distribution in the separation region:

$$\frac{d^2c}{dy^2} - b\frac{dc}{dy} = 0, \tag{2}$$

where

$$y = \frac{Hz}{K}; \quad H = \frac{\alpha\rho^2 g\beta\delta^3 (\Delta T)^2}{6!\eta\bar{T}}; \quad K = \frac{g^2\rho^3\beta^2\delta^7 (\Delta T)^2 B}{9!\eta^2 D}. \tag{3}$$

There are changes in the contents of the target component in the upper channel $\sigma_e dc_e$ and the lower one $\sigma_i dc_i$ in the part dx because of the transport $\tau dx/B$ through the separating region in this part, so when one incorporates the opposite flow directions, one has

$$\left.\frac{\tau}{B\sigma_e} dx\right|_{y=y_e} = dc_e, \quad \left.\frac{\tau}{B\sigma_i}\right|_{y=0} = dc_i. \tag{4}$$

One solves (2) with (4) to get [4] the following expressions for the concentration distributions in the two channels:

$$c_e = D_1 e^{\varphi\xi} + \left[D_1\left(\frac{b}{\varphi\varkappa_i} - 1\right)e^{\varphi\xi} + D_2\right]\exp(by_e) - \frac{a}{b}, \tag{5}$$

$$c_i = D_1\frac{b}{\varphi\varkappa_i}e^{\varphi\xi} - \frac{a}{b} + D_2, \tag{6}$$

where

$$\varphi = \frac{b\exp(by_e) - \dfrac{\varkappa_i}{\varkappa_e}}{\varkappa_i\exp(by_e) - 1}; \quad \varkappa_e = \frac{\sigma_e}{H}; \quad \varkappa_i = \frac{\sigma_i}{H}; \quad \xi = \frac{x}{B}. \tag{7}$$

To determine D_1 and D_2, we incorporate the features of mixture and output, which govern the conditions at the ends. In this case (Fig. 1a),

$$c_e|_{\xi=0} = c_0, \quad c_e|_{\xi=1} = c_i|_{\xi=1}. \tag{8}$$

Then (5) and (6) with (8) give

$$D_1 = \left(c_0 + \frac{a}{b}\right) \Big/ \left(1 + \left(\frac{b}{\varphi\varkappa_i} - 1\right)(1 - e^{\varphi})\exp(by_e)\right), \tag{9}$$

$$D_2 = \left(c_0 + \frac{a}{b}\right)\left(1 - \frac{b}{\varphi\varkappa_i}\right)e^{\varphi} \Big/ \left(1 + \left(\frac{b}{\varphi\varkappa_i} - 1\right)(1 - e^{\varphi})\exp(by_e)\right). \tag{10}$$

We substitute (9) and (10) into (5) and (6) to get formulas for the concentration distributions:

$$c_e + \frac{a}{b} = \left(c_0 + \frac{a}{b}\right) \frac{1 + \left(\frac{b}{\varphi \varkappa_i} - 1\right)(1 - e^{\varphi - \varphi\xi}) \exp(by_e)}{1 + \left(\frac{b}{\varphi \varkappa_i} - 1\right)(1 - e^{\varphi}) \exp(by_e)} e^{\varphi\xi}, \quad (11)$$

$$c_i + \frac{a}{b} = \left(c_0 + \frac{a}{b}\right) \frac{e^{\varphi} + \frac{b}{\varphi \varkappa_i}(e^{\varphi\xi} - e^{\varphi})}{1 + \left(\frac{b}{\varphi \varkappa_i} - 1\right)(1 - e^{\varphi}) \exp(by_e)}. \quad (12)$$

One also incorporates the balance relations for the total amount of mixture

$$\sigma_0 = \sigma_i + \sigma_p \quad (13)$$

and for the extracted component

$$c_0 \sigma = \sigma_i c_i + \sigma_p c_{ek}. \quad (14)$$

The concentration of the component c_{ik} in the reject flow is restricted by the condition for equilibrium at $\tau = 0$ with the incoming flow c_0, which gives [6]

$$\frac{a + bc_0}{a + bc_{ik}^*} = \exp(by_e) = \alpha, \quad (15)$$

where c_{ik}^* is the equilibrium concentration.

The approach to the equilibrium concentration in the reject flow is [7] defined by

$$\psi = \frac{c_{ik} - c_{ik}^*}{c_0 - c_{ik}^*}. \quad (16)$$

Then (12) and (16) give

$$\psi = 1 \Big/ \left(1 + \left(\frac{b}{\varphi \varkappa_i} - 1\right)(1 - e^{\varphi}) \exp(by_e)\right). \quad (17)$$

Then (17) and (11) with $\xi = 1$ give an expression for the exit concentration:

$$\frac{c_{ek} + \frac{a}{b}}{c_0 + \frac{a}{b}} = e^{\varphi}\psi. \quad (18)$$

Subsequently, the total amount of mixture σ_0 entering the apparatus is taken as constant and we use the ratio of the product flow to the total amount of mixture k. We have $\sigma_0 = \sigma_e$ for flow closure and from (13) and (14)

$$k = \frac{\sigma_p}{\sigma_e} = \frac{c_0 - c_{ik}}{c_{ek} - c_{ik}}. \quad (19)$$

The analysis of (18) is substantially simplified if we use the ratio of the actual throughput to the maximal corresponding to the equilibrium concentration together with the maximal k:

$$\theta = \frac{\sigma_p}{\sigma_{p\max}}, \quad k_{\max} = \frac{\sigma_{p\max}}{\sigma_e} = \frac{k}{\theta} = \frac{c_0 - c_{ik}^*}{c_{ek} - c_{ik}^*}. \quad (20)$$

From (7), (13)-(16), (19), and (20) we have

$$\varphi = \frac{b}{\varkappa_p} \frac{k}{1-k}\left(1 + \frac{k}{\alpha - 1}\right), \quad \psi = \frac{1-\theta}{1-k}. \quad (21)$$

Then (21) with (18) gives a formula for y_e/κ_p, which defines the area of the apparatus per unit product:

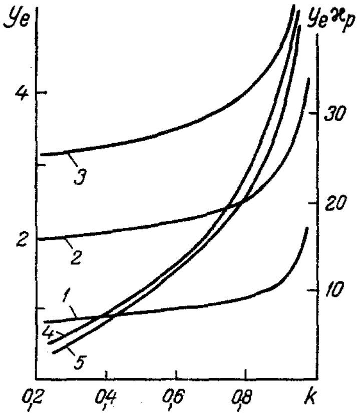

Fig. 2. Diagram for choosing optimal apparatus parameters with partial mixture recycling: 1) $y_e/\kappa_p = f_1(k)$, $q_e = 10$; 2) $y_e/\kappa_p = f_2(k)$, $q_e = 100$; 3) $y_e/\kappa_p = f_3(k)$, $q_e = 1000$; 4) $y_e = \phi_1(k)$, $q_e = 100$, $q_e = 1000$, 5) $y_e = \phi_2(k)$, $q_e = 10$.

$$\frac{y_e}{\varkappa_p} = \frac{y_e}{b}\frac{1-k}{k(1+k/(\alpha-1))}\ln\left(q_e\frac{1-\theta}{1-k}\right). \tag{22}$$

In removing a minor impurity, where a = 1 and b = −1, we have from (15) and (19) that

$$\alpha = \exp(-y_e),\ k_{\max} = q_e\frac{1-\alpha}{q_e-\alpha},\quad q_e = \frac{1-c_0}{1-c_e}, \tag{23}$$

and (23) becomes

$$\frac{y_e}{\varkappa_p} = y_e\frac{1-k}{k(1+k/(\alpha-1))}\ln\left(q_e\frac{1-\theta}{1-k}\right). \tag{24}$$

It follows from (24) that the working area per unit product in unit time is dependent on k for given q_e and y_e; calculations show that each k corresponds to a y_e for which y_e/κ_p is minimal. Figure 2 shows curves for choosing these parameters, which is derived from (24) for some q_e. The diagram can be used with a given k to determine y_e and the corresponding minimal y_e/κ_e. The optimum y_e is almost independent of q_e for $q_e \geq 100$, while as y_e decreases, which determines the height, one requires a smaller area per unit product. However, k decreases at the same time, which leads to an increase in the amount of discarded mixture.

NOTATION

α, thermal-diffusion constant; β, bulk expansion coefficient; δ, distance between heated and cooled surfaces; ρ, density, σ, mass rate of flow through channel; η, dynamic viscosity; B, apparatus length; T_2 and T_1, temperatures of the heated and cooled surfaces; $\Delta T = T_2 - T_1$; $\bar{T} = {}^1/_2(T_2 - T_1)$; z, vertical coordinate; x, longitudinal coordinate; c, mass concentration. Subscripts: e, upper channel, i, lower channel, 0, initial value, k, exit from apparatus.

LITERATURE CITED

1. D. Frazier, Thermal Diffusion Method and Apparatus, USA Patent No. 2827172, March 18 (1958).
2. D. Frazier, Ind. Chem. Process Design Devel., 1, No. 4, 237-241 (1962).
3. G. D. Rabinovich, Inzh.-Fiz. Zh., 31, No. 3, 514-522 (1976).
4. A. V. Suvorov and G. D. Rabinovich, Inzh.-Fiz. Zh., 41, No. 2, 231-238 (1981).
5. A. V. Suvorov and G. D. Rabinovich, Inzh.-Fiz. Zh., 39, No. 1, 86-95 (1980).
6. G. D. Rabinovich, Separating Isotopes and Other Mixtures by Thermal Diffusion [in Russian], Moscow (1981).
7. A. M. Rozen, The Theory of Isotope Separation in Columns [in Russian], Moscow (1960).